

Bridge Structure Impact Modelling

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ABSTRACT: The response of single storey structures subject to earthquake loading in which impact may occur between the structures is investigated. These structures represent bents of a bridge which may have different masses strengths and stiffness and an expansion joint where deformation may occur. The likelihood of increase in response due to impact was quantified in a probabilistic sense for different normalized distances between the structures. The increase of displacement relative to the damage on the structure is computed for ranges of periods. The effect of including a high-force-to-volume (HF2V) lead dissipater between the parts of the bridge is then considered and it is shown that the device does not significantly affect the response in this case.

1 INTRODUCTION

Expansion joints between bridge sections help mitigate internal forces caused by thermal expansion and contraction. However, these gaps can lead to relative displacements and create impact or pounding hazards during earthquakes as a result of increased demands and damage resulting from collisions between different parts of the structure. Dramatic examples of catastrophic pounding damage have occurred between adjacent buildings in highly urbanized areas as shown in the 1985 Mexico and the 1995 Kobe earthquakes. A number of studies have been conducted to look at pounding effects. Many of these have been summarised by Cole (2010), Dimitrakopoulos et al. (2009) and Chouw and Hao (2009), but information is generally not available to answer the following questions:

- 1) For oscillators a certain distance apart, what is the probability of impact over suites of probabilistically scaled ground motions? What is the influence of the oscillators periods?
- 2) What is the probability that displacements will increase as a result of impact?
- 3) How far apart should structures be placed to minimize or avoid increased displacements from impact?

In addition, the use of novel seismic dissipaters, such as the HF2V dissipaters (Rodgers et al. 2008), may possibly influence the effect of the response, and this leads to a fourth question:

- 4) What is the effect of lead dissipaters placed between the decks?

This paper describes in initial study used to study these questions to develop answers in a useful form for designers.

2 MODELLING

To model impact, two single-degree-of-freedom oscillations are used, as shown in Figure 1. The oscillators are decoupled with their own mass, stiffness, damping, and hence period. Both oscillations have the same height so collisions only occur between the structures at the floor level. They are subjected to the same seismic ground motions because they are located close to each other. The gap distance, λ , must be greater than $x_1 - x_2$ if impact does not occur, where x_1 and x_2 are the displacements of the oscillators. For example, if $\lambda = 100\text{mm}$, and $x_1 = 80\text{mm}$, and $x_2 = -20\text{mm}$, then impact will just occur. For larger λ , impact will not occur.

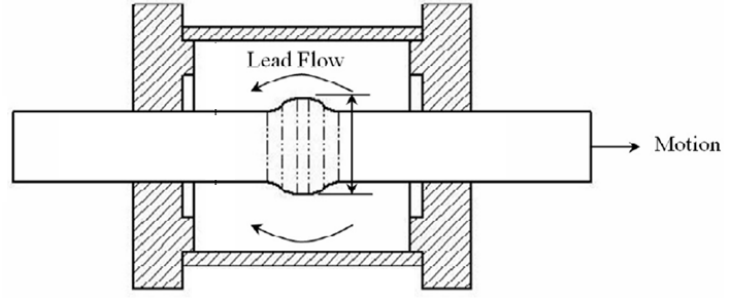
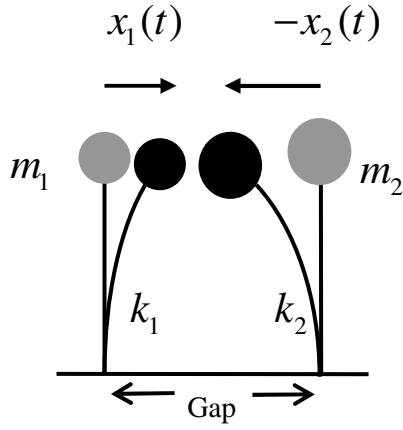


Figure 1. Oscillator used for impact study Figure 2. Bulged HF2V device (Rodgers et al. 2008)

Pre and post impact behaviour occurs when $x_1 < x_2 + \lambda$. Equations 1a and 1b modelling this response where k_1 and k_2 are the oscillator stiffnesses, m_1 and m_2 are their masses as defined in Figure 1. In this case the system is decoupled. The Newmark-beta constant average acceleration numerical integration is utilized to model these responses.

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_{eq} \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = -m_2 \ddot{x}_{eq} \end{cases} \quad (1)$$

When impact occurs the two masses touch. Stereomechanics can be used to model the response (Cole 2010). A coefficient of restitution, e , can be used to describe the energy dissipated in the collision. If no energy is dissipated in heat or permanent deformation, then the collision is fully elastic and $e = 1$. A coefficient of restitution equal to zero is for a fully plastic collision. In reality, the coefficient of restitution is sensitive to the prior-impact velocity and tends to range between 0.65 and 0.8 (Jankowski, 2010). Conservation of momentum and kinetic energy enables the post-impact velocities of the oscillators to be calculated according to Equation 2 where \dot{x}_1, \dot{x}_2 are the velocities before the impact, and \dot{x}'_1 and \dot{x}'_2 are just after impact. In this pilot study, $e = 1$ is used. This is generally conservative.

$$\dot{x}'_1 = \dot{x}_1 - (1+e) \frac{m_2}{m_1 + m_2} (\dot{x}_1 - \dot{x}_2) \quad \dot{x}'_2 = \dot{x}_2 + (1+e) \frac{m_1}{m_1 + m_2} (\dot{x}_1 - \dot{x}_2) \quad (2)$$

Behaviour during impact the impact force, F_{inc} , over a timestep, dt , can be found from conservation of momentum according to Equation 3.

$$m(\dot{x}' - \dot{x}) = F_{inc} dt \quad (3)$$

So the equations of motion are coupled when impact occurs ($x_1 \geq x_2 + \lambda$) and are defined:

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_{eq} - F_{inc} \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = -m_2 \ddot{x}_{eq} + F_{inc} \end{cases} \quad (4)$$

The response can be normalized by defining a gap ratio, GR , as the gap, λ (or GAP), divided by the sum of the spectral displacements of both oscillators, S_{d1} and S_{d2} . If $GR \geq 1.0$ then there is no possibility of collision. The probability of impact increases as the gap ratio decreases.

$$GR = \frac{GAP}{S_{d1} + S_{d2}} \quad (5)$$

The analyses described in this paper used a mass ratio (m_1/m_2) equal to 1, period ratios (T_1/T_2) of 0.1, 0.5, 1.5 and 10, and an initial stiffness proportional damping ratio of 5%. The structures were considered to respond elastically.

The earthquake records utilized are three Los Angeles suites from the SAC Steel Project. The low, medium and high suites represent ground motions having probabilities of exceedance of 50% in 50 years, 10% in 50 years and 2% in 50 years [19]. Hence, these suites represent equally likely events spanning a range of likelihood of occurrence, and thus a range of magnitudes. Each record was run in both directions to remove directional effects. Gap ratios ranged from 0.1 to 1.0 in increments of 0.1.

The following were obtained for each mass ratio over each suite (i.e. low, medium and high) of ground motions:

- Minimum gap ratio for less than a specified probability (i.e. 10%, 20%, or 50%) of increase for a specified percentage (i.e. 50% or 90%) of earthquakes.
- Probability of impact for gap ratios of 0.5, 0.8 and 0.9 over a range of (T_1 , T_2) values and a mass ratio of 1 (Table 3).

The HF2V device is a passive actuator, utilizing lead-extrusion to dissipate energy as shown in Figure 2. Yielding of the lead dissipates kinetic energy from the bulged shaft into strain energy and heat within the actuator. The elastic-plastic deformation of the lead exhibited by this actuator produces an almost ideal square hysteretic loop as shown experimentally by Rodgers et al. (2008). The stiffness of the devices is sensitive to their connections to the frame. To numerically consider this effect in the analyses an expression for the device force output as a function of both the shaft stiffness and relative velocity between two frames. The device force F_D is hence calculated using Equations (6) and (7) where $\Delta x = (x_2 - x_1)$, the relative displacement between the masses; y is the displacement across the device; Δt = numerical integration time step; C_α = the device coefficient; α = the damper velocity exponent kept constant at 0.012 (Rodgers et al. 2008, 2009, 2010); f_D = flexibility of the connecting shaft or $(1/K)$ where K is the effective stiffness of the connections taken as EA/L where E is the shaft elastic modulus, A is the area of the shaft and L is the shaft length. The HF2V device was designed using the procedure of Rodgers et al. (2008) with a nominal shaft diameter of 30mm.

$$\dot{y}_{i+1} = \frac{\Delta x_{i+1} - \Delta x_i + f_D F_i}{\Delta t + f_D C_\alpha \left(\frac{\dot{y}_i}{\alpha}\right) |F_{i+1}|^{1-1/\alpha}} \quad (6)$$

$$F_{i+1} = C_\alpha |\dot{y}_{i+1}|^\alpha \text{sgn}(\dot{y}_{i+1}) \quad (7)$$

Assuming a length of 0.5m, the flexibility, f_D is 3.4×10^{-9} m/N. The device strength is related to the device coefficient C_α . In this study, C_α is calculated as per Equation 8 as a function of the seismic weight of the structure to which it is attached.

$$C_\alpha = \frac{M_{\text{fraction}} W}{\alpha_{\text{ref}}^\alpha}; \quad W = M_1 g \quad (8)$$

HF2V dampers are sized according to relative mass fraction, typical values ranging in the order of 1-10% of the smaller of the masses (Trochalakis et al., 2000). For the purpose of this study, the masses of both frames are identical, and kept constant. Hence, using a set value of mass fraction of the device models a damper of set characteristics across all simulations. A model showing the bridge columns, the gapping, the energy dissipation and the system is given in Figure 3.

3 DYNAMIC BEHAVIOUR

Figure 4 shows the responses of two oscillators with impact for one record. Here there are 4 major impacts, and the 2nd and 4th impacts seem to occur at the high velocity causing the oscillators bounce off each other.

Since the oscillators move by different amounts in different directions with different phases, a gap ratio less than one does not always imply that there will be impact. This is shown in Table 1 which

shows the probability of at least one impact. It may be seen that the probability of impact increases for smaller gap ratios. Also, there is no impact when $T_1 = T_2$ because the oscillators are in phase. In general, the probability increases as the T_1 becomes more different from T_2 . It may also be seen that the tables are symmetrical about the diagonal. This is because earthquake records in both directions were considered.

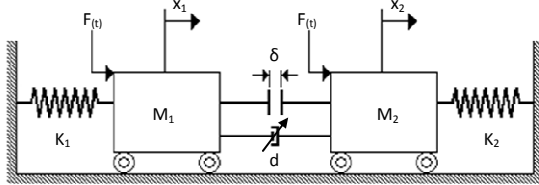


Figure 3. Mechanical model of System (Kujikis, 2010)

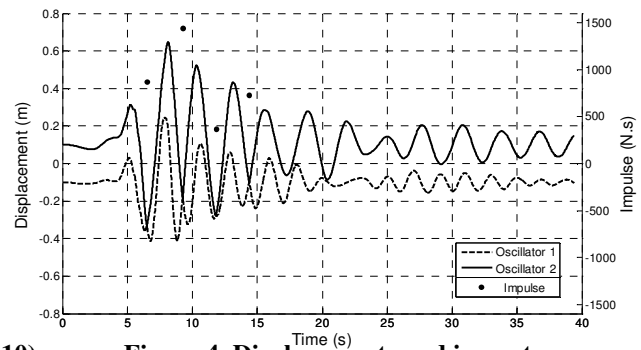


Figure 4. Displacements and impact
($T_1=2s$, $T_2=3s$, $M_1/M_2=1$, $e = 1$, $\zeta = 0.05$, LA04)

Table 1. Probability of at least one impact (60 earthquakes in both directions, $M_1=M_2=1000kg$, $\zeta = 0.05$)

GR=0.5							
T2 \ T1	0,2	0,5	0,8	1	3	5	
0,2	0,00	0,98	1,00	1,00	1,00	0,99	
0,5	0,98	0,00	0,98	0,99	0,99	0,98	
0,8	1,00	0,98	0,00	0,98	1,00	0,99	
1	1,00	0,99	0,98	0,00	0,98	0,99	
3	1,00	0,99	1,00	0,98	0,00	0,88	
5	0,99	0,98	0,99	0,99	0,88	0,00	

GR=0.8							
T2 \ T1	0,2	0,5	0,8	1	3	5	
0,2	0,00	0,62	0,83	0,85	0,88	0,82	
0,5	0,61	0,00	0,22	0,39	0,69	0,73	
0,8	0,82	0,22	0,00	0,02	0,52	0,63	
1	0,85	0,39	0,02	0,00	0,42	0,50	
3	0,88	0,69	0,52	0,42	0,00	0,24	
5	0,82	0,73	0,63	0,50	0,24	0,00	

GR=0.9							
T2 \ T1	0,2	0,5	0,8	1	3	5	
0,2	0,00	0,13	0,48	0,51	0,65	0,65	
0,5	0,13	0,00	0,03	0,09	0,37	0,41	
0,8	0,48	0,03	0,00	0,00	0,14	0,23	
1	0,51	0,09	0,00	0,00	0,13	0,22	
3	0,65	0,37	0,14	0,13	0,00	0,04	
5	0,65	0,41	0,23	0,22	0,04	0,00	

Table 2 shows the minimum gap ratios to provide a specified level of confidence that the increase in response of an oscillator is less than a specified amount. This has been computed for different T_1 and T_2 . It may be seen again that when $T_1 = T_2$ there is not impact, and greater increases tend to occur when as the periods differ. It may be seen that the left and right tables are transposes of each other

because earthquake waves are considered from both directions. The analyses shown are for a mass ratio of 1.0, but have also been conducted for other mass ratios (e.g. Labrosse, 2010). The significance of the values in the table can be described by an example.

Consider two bridges with a mass ratio of 1.0, and periods T_1 of 1.0s and $T_1 = 0.2$ s due to the different stiffnesses of the columns beneath them. The resulting spectral displacements for these periods are 200mm and 60mm from the design level response spectra considered. To ensure an increase of no more than 10% in 50% of the time, the required gap ratio must be 0.80. This means a gap of 208mm is required. To have an increase of no more than 10% with 90% confidence, the required gap ratio must be 0.90. This means a gap of 234mm is required. As expected, a greater gap is needed to have a higher degree of confidence that there will be less than a 10% increase in displacement. The table can also be used with a higher degree of confidence required for a design level earthquake than for a maximum credible earthquake. This type of information is useful for designers.

For a specific gap, the likely change in response can also be obtained. This is found for the medium suite of records based on Table 3. Using the example above, with a mass ratio of 1.0, and periods T_1 of 1.0s and $T_1 = 0.2$ s, Oscillator 1 displacement will increase by a median value of 51% from 200mm to 302mm, but Oscillator 2 displacement decreases by a median value of 4% from 60mm to 57.6mm.

4 EFFECT OF LEAD DISSIPATORS

The effectiveness of a HF2V dissipater in reducing the structural response of the bridge was investigated. It was considered that by placing the dissipater/damper between the ground and the structure the response was sure to significantly decrease. Therefore, in this study the dissipater was placed between the masses of the two oscillators. This is perhaps an unusual location to place the oscillator, however, it was considered to be interesting. However, the response of Frame 2, which is given periods from 0.1 and 3.0 seconds in 0.1 second steps is not very significant as shown in Figure 5. The period of Frame 1 was kept constant at 0.8s. It may be seen that the dissipater became most effective when the periods of the frames were most dissimilar.

5 CONCLUSIONS

A study was conducted to evaluate the effect of impact on the response of bridges structures. An impact coefficient of restitution of unity was conservatively considered. It was found that:

1. A gap ratio is defined to normalize the gap between masses with spectral displacements of oscillators. The probability of impact was quantified for a range of oscillators with different gap ratios, periods, and masses. The probability tended to increase in the oscillator with longer period and decrease in the oscillator with shorter period. This tendency occurred with greater period differences. When the periods of both oscillators were identical, there was no impact.
2. The gap ratio to ensure that the response did not increase by more than a specified percentage was quantified with a specified level of confidence was quantified. An example was provided on how to use this information.
3. For a specific gap ratio, the median increase in response was quantified.
4. A lead energy dissipater with a strength up to 20% of the weight of one oscillator was placed between the two structures. The dissipater effect was most significant when the frames had different periods, however, the effect was still relatively small and did not significantly reduce the peak displacement responses.

Table 2. Minimum gap ratios for different cases at increased displacement at mass 1 (left) and mass 2 (right) for a given percentage of the ground motions analysed in both directions (mass ratio= 1, $e = 1$, $\zeta = 0.05$)

Minimum Gap Ratio for less than 10% of Increase of displacement of mass 1(left) and mass 2(right) for 50% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.70	0.75	0.80	0.83	0.80	0.2	0.00	0.00	0.00	0.00	0.00	0.40
0.5	0.00	0.00	0.10	0.33	0.18	0.28	0.5	0.70	0.00	0.00	0.00	0.05	0.58
0.8	0.00	0.00	0.00	0.03	0.08	0.05	0.8	0.75	0.10	0.00	0.00	0.05	0.50
1	0.00	0.00	0.00	0.00	0.00	0.03	1	0.80	0.33	0.03	0.00	0.05	0.45
3	0.00	0.05	0.05	0.05	0.00	0.00	3	0.83	0.18	0.08	0.00	0.00	0.50
5	0.40	0.58	0.50	0.45	0.50	0.00	5	0.80	0.28	0.05	0.03	0.00	0.00

Minimum Gap Ratio for less than 10% of Increase of displacement of mass 1 (left) and mass 2 (right) for 90% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.80	0.90	0.90	0.90	0.90	0.2	0.00	0.06	0.01	0.01	0.53	0.90
0.5	0.06	0.00	0.56	0.66	0.81	0.81	0.5	0.80	0.00	0.06	0.30	0.67	0.86
0.8	0.01	0.06	0.00	0.21	0.75	0.66	0.8	0.90	0.56	0.00	0.51	0.66	0.86
1	0.01	0.30	0.51	0.00	0.61	0.66	1	0.90	0.66	0.21	0.00	0.66	0.81
3	0.53	0.67	0.66	0.66	0.00	0.27	3	0.90	0.81	0.75	0.61	0.00	0.76
5	0.90	0.86	0.86	0.81	0.76	0.00	5	0.90	0.81	0.66	0.66	0.27	0.00

Minimum Gap Ratio for less than 20% of Increase of displacement of mass 1 (left) and mass 2 (right) for 50% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.65	0.75	0.78	0.83	0.78	0.2	0.00	0.00	0.00	0.00	0.00	0.25
0.5	0.00	0.00	0.00	0.15	0.10	0.23	0.5	0.65	0.00	0.00	0.00	0.03	0.43
0.8	0.00	0.00	0.00	0.00	0.00	0.00	0.8	0.75	0.00	0.00	0.00	0.00	0.50
1	0.00	0.00	0.00	0.00	0.00	0.00	1	0.78	0.15	0.00	0.00	0.00	0.43
3	0.00	0.03	0.00	0.00	0.00	0.00	3	0.83	0.10	0.00	0.00	0.00	0.48
5	0.25	0.43	0.50	0.43	0.48	0.00	5	0.78	0.23	0.00	0.00	0.00	0.00

Minimum Gap Ratio for less than 20% of Increase of displacement of mass 1 (left) and mass 2 (right) for 90% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.80	0.86	0.90	0.90	0.90	0.2	0.00	0.01	0.00	0.00	0.53	0.90
0.5	0.01	0.00	0.41	0.61	0.80	0.76	0.5	0.80	0.00	0.00	0.08	0.67	0.86
0.8	0.00	0.00	0.00	0.16	0.71	0.61	0.8	0.86	0.41	0.00	0.22	0.66	0.81
1	0.00	0.08	0.22	0.00	0.57	0.62	1	0.90	0.61	0.16	0.00	0.65	0.81
3	0.53	0.67	0.66	0.65	0.00	0.17	3	0.90	0.80	0.71	0.57	0.00	0.71
5	0.90	0.86	0.81	0.81	0.71	0.00	5	0.90	0.76	0.61	0.62	0.17	0.00

Minimum Gap Ratio for less than 50% of Increase of displacement of mass 1 (left) and mass 2 (right) for 50% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.50	0.65	0.75	0.75	0.68	0.2	0.00	0.00	0.00	0.00	0.00	0.13
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.5	0.50	0.00	0.00	0.00	0.00	0.13
0.8	0.00	0.00	0.00	0.00	0.00	0.00	0.8	0.65	0.00	0.00	0.00	0.00	0.23
1	0.00	0.00	0.00	0.00	0.00	0.00	1	0.75	0.00	0.00	0.00	0.00	0.25
3	0.00	0.00	0.00	0.00	0.00	0.00	3	0.75	0.00	0.00	0.00	0.00	0.00
5	0.13	0.13	0.23	0.25	0.00	0.00	5	0.68	0.00	0.00	0.00	0.00	0.00

Minimum Gap Ratio for less than 50% of Increase of displacement of mass 1 (left) and mass 2 (right) for 90% of earthquakes													
T2 \ T1	0.2	0.5	0.8	1	3	5	T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.75	0.85	0.90	0.90	0.90	0.2	0.00	0.00	0.00	0.00	0.19	0.81
0.5	0.00	0.00	0.12	0.51	0.66	0.65	0.5	0.75	0.00	0.00	0.00	0.11	0.81
0.8	0.00	0.00	0.00	0.00	0.47	0.56	0.8	0.85	0.12	0.00	0.00	0.44	0.80
1	0.00	0.00	0.00	0.00	0.41	0.34	1	0.90	0.51	0.00	0.00	0.46	0.81
3	0.19	0.11	0.44	0.46	0.00	0.00	3	0.90	0.66	0.47	0.41	0.00	0.60
5	0.81	0.81	0.80	0.81	0.60	0.00	5	0.90	0.65	0.56	0.34	0.00	0.00

Table 3. Median of displacement change (gap ratios=(0.2, 0.5, 0.8), $M_1=M_2=1000\text{kg}$, $e=1$, $\zeta=0.05$, 20 earthquakes records of the middle suite in both directions)

GR=0.2

oscillator 1

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.83	1.27	1.48	1.61	2.09
0.5	-0.27	0.00	0.07	0.17	0.17	0.22
0.8	-0.25	-0.15	0.00	0.00	0.01	0.09
1	-0.24	-0.08	0.03	0.00	0.01	0.02
3	-0.17	0.02	-0.02	0.02	0.00	-0.11
5	0.56	0.58	0.65	0.97	0.52	0.00

oscillator 2

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	-0.27	-0.25	-0.24	-0.17	0.56
0.5	0.83	0.00	-0.15	-0.08	0.02	0.58
0.8	1.27	0.07	0.00	0.03	-0.02	0.65
1	1.48	0.17	0.00	0.00	0.02	0.97
3	1.61	0.17	0.01	0.01	0.00	0.52
5	2.09	0.22	0.09	0.02	-0.11	0.00

GR=0.5

oscillator 1

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.78	1.19	1.51	1.67	1.63
0.5	-0.18	0.00	0.00	0.06	0.10	0.22
0.8	-0.17	-0.03	0.00	0.00	0.01	0.00
1	-0.15	-0.01	0.00	0.00	0.01	0.00
3	-0.01	0.03	0.09	0.05	0.00	-0.06
5	0.50	0.65	0.66	0.51	0.42	0.00

oscillator 2

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	-0.18	-0.17	-0.15	-0.01	0.50
0.5	0.78	0.00	-0.03	-0.01	0.03	0.65
0.8	1.19	0.00	0.00	0.00	0.09	0.66
1	1.51	0.06	0.00	0.00	0.05	0.51
3	1.67	0.10	0.01	0.01	0.00	0.42
5	1.63	0.22	0.00	0.00	-0.06	0.00

GR=0.8

oscillator 1

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	0.00	0.20	0.51	0.90	0.60
0.5	-0.01	0.00	0.00	0.00	0.00	0.00
0.8	-0.03	0.00	0.00	0.00	0.00	0.00
1	-0.04	0.00	0.00	0.00	0.00	0.00
3	-0.01	0.00	0.00	0.00	0.00	0.00
5	0.11	0.07	0.09	0.00	0.00	0.00

oscillator 2

T2 \ T1	0.2	0.5	0.8	1	3	5
0.2	0.00	-0.01	-0.03	-0.04	-0.01	0.11
0.5	0.00	0.00	0.00	0.00	0.00	0.07
0.8	0.20	0.00	0.00	0.00	0.00	0.09
1	0.51	0.00	0.00	0.00	0.00	0.00
3	0.90	0.00	0.00	0.00	0.00	0.00
5	0.60	0.00	0.00	0.00	0.00	0.00

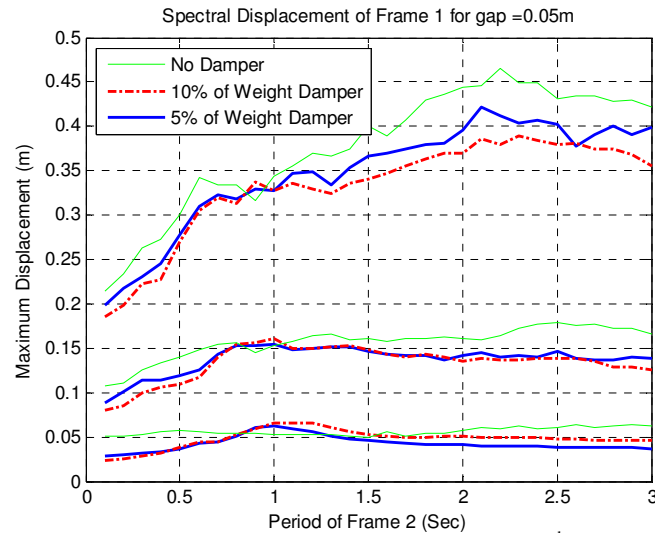


Figure 5. Effect of HF2V Damper on the Peak Response – 16th, 50th and 84th percentiles

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